SURFACE COMPLETION OF SHAPE AND TEXTURE BASED ON ENERGY MINIMIZATION

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ABSTRACT

In this paper, we propose a novel surface completion method to generate plausible shapes and textures for missing regions of 3D models. The missing regions are filled in by minimizing two energy functions for shape and texture, which are both based on similarities between the missing region and the rest of the object; in doing so, we take into account the positive correlation between shape and texture. We demonstrate the effectiveness of the proposed method experimentally by applying it to two models.

***Index Terms*—** Surface Completion, Shape and Texture, Energy Minimization

1. INTRODUCTION

Surface completion is a technique for filling in missing regions in 3D models caused by sensing errors and occlusions in measuring target objects. Surface completion methods can be broadly classified into two categories: one uses information around missing regions and the other uses exemplars in the rest of the object, referred to as data region. The former methods [1–4] fill in missing regions with smooth surface patches using partial differential equations or signed distance field. These methods are effective for small gaps but cannot generate complex shapes. Therefore, the latter methods such as [5–10] have recently been developed so as to generate complex shapes in missing regions.

A number of surface completion algorithms in recent years have dealt with both shape and color, rather than shape alone [6–8]. These methods calculate the similarities of shape and color between missing and data regions, and generate complex shape and texture by copying the similar patch to the missing region. For instance, in [6], one similar patch whose size is almost the same as a missing region is required to exist in the model. In [7, 8], similar local patches are successively copied to the missing region from the outer to the inner part; as such, a discontinuous surface may be generated on the seam in the completed model. Also, sample shape and color are selected from the same position in these methods. While there is correlation between shape and color of many objects, similar shapes do not necessarily always have the same texture. Therefore, unnatural shape and color could potentially be generated due to the limited available exemplars.

In this paper, we propose a surface completion method by taking into account shape and texture simultaneously. Specifically, we minimize two energy functions for shape and texture by extending our previous energy function in [9, 10], which is only based on the shape similarity between missing and data regions. In this paper, shape and color in missing regions are determined as a global solution by minimizing the two energy functions in an iterative fashion. In addition, sample shapes and textures can be selected from different positions, while the positive correlation between shape and texture is taken into account. Target models in this research consist of vertices, triangles and texture images where the correspondence of coordinates between vertices and texture images is assumed to be known.

2. PROPOSED APPROACH

Our surface completion process can be described as follows: (a) the user manually selects missing regions to be repaired in a model; (b) initial position and color are assigned to vertices and triangles in the missing regions; (c) the position and color of vertices in the missing region are updated iteratively so as to minimize the two energy functions. Section 2.1 describes the energy function, and Section 2.2 describes the minimization of the energy functions.

2.1. Energy functions for shape and texture similarity

As illustrated in Fig. 1, a 3D model is first divided into region Ω' which includes missing region Ω selected by the user, and data region Φ which is the remainder of the object. Ω' is determined so that spherical patch $A_{\mathbf{p}_i}$ with radius l and central vertex $\mathbf{p}_i \in \Omega'$, includes at least one vertex in region Ω. Two energy functions, E_s for shape and E_t for texture, are defined based on the weighted sum of shape similarity SSD_s and texture similarity SSD_t between vertices in region Ω' and

*Rsearch Fellow supported by the Ministry of Education, Science, Sports and Culture, Grant-in-Aid for JSPS Fellows, 21-8045.

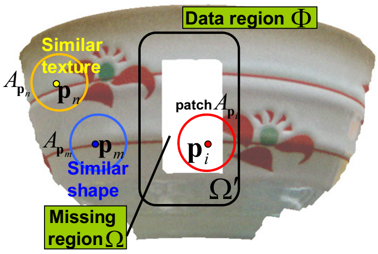


Fig. 1. Missing and data regions in a 3D model.

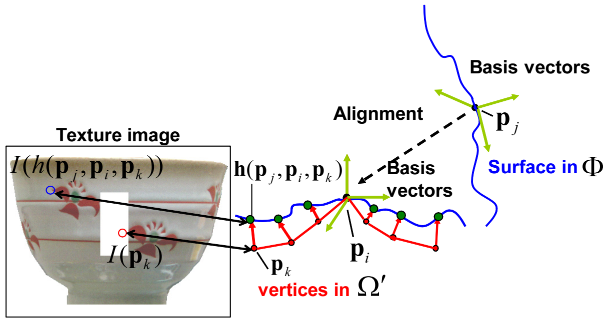


Fig. 2. Alignment of vertices and surface for similarity measure of shape and texture.

surface in data region Φ. In doing so, the positive correlation between shape and texture is taken into account as follows:

$$E_s = \sum_{\mathbf{p}_i \in \Omega'} w_{\mathbf{p}_i} (SSD_s(\mathbf{p}_i, \mathbf{p}_m) + \alpha SSD_t(\mathbf{p}_i, \mathbf{p}_m)) / \sum_{\mathbf{p}_i \in \Omega'} w_{\mathbf{p}_i}, \tag{1}$$

$$E_t = \sum_{\mathbf{p}_i \in \Omega'} \lambda_{\mathbf{p}_i} (SSD_t(\mathbf{p}_i, \mathbf{p}_n) + \beta SSD_s(\mathbf{p}_i, \mathbf{p}_n)) / \sum_{\mathbf{p}_i \in \Omega'} \lambda_{\mathbf{p}_i}, \tag{2}$$

In the above energy functions, there are four kinds of variables: vertex's 3D position $\mathbf{p}_i$, intensity $I(\mathbf{p}_i)$ of vertex $\mathbf{p}_i$, position $\mathbf{p}_m$ of similar shape, and position $\mathbf{p}_n$ of similar texture in region Φ corresponding to vertex $\mathbf{p}_i$. Weight $w_{\mathbf{p}_i}$ is set to 1 if $\mathbf{p}_i \in \Omega' \cap \overline{\Omega}$ because the positions of vertices in this region are known and fixed; otherwise $w_{\mathbf{p}_i}$ is set to c^{-d} where d is the minimum number of links from all vertices on the boundary of Ω to vertex $\mathbf{p}_i$, and c is a positive constant; this is because the confidence level is higher for position of vertices near the boundary than those in the center of the missing region. Weight $\lambda_{\mathbf{p}_i}$ is defined in a similar way as $w_{\mathbf{p}_i}$. Weights α and β are the relative importance of shape versus texture similarity in searching for similar shape and texture. The minimization is done with respect to all the vertices in Ω. Eqs. (1) and (2) are normalized with respect to sum of weights because the weight of each vertex changes in the energy minimization process. In the following, the similarity measures SSD_s and SSD_t are described in detail.

As shape similarity SSD_s, we employ the same measure as our previous method [10]. Concretely, the surface around $\mathbf{p}_j$ in data region Φ is aligned to vertices around $\mathbf{p}_i$ in Ω' so that 3D position and basis vectors of vertex $\mathbf{p}_j$ overlap those of vertex $\mathbf{p}_i$ as shown in Fig. 2. The basis vectors are determined using directions of normal and principal curvatures [10]. We refer to the normal basis vector of a vertex as its "normal vector". $SSD(\mathbf{p}_i, \mathbf{p}_j)$ is computed as the sum of distances between vertices in spherical patch $A_{\mathbf{p}_i}$ with radius l and central vertex $\mathbf{p}_i \in \Omega'$, and the aligned surface around $\mathbf{p}_j \in \Phi$ as follows:

$$SSD_s(\mathbf{p}_i, \mathbf{p}_j) = \sum_{\mathbf{p}_k \in A_{\mathbf{p}_i}} \frac{\| \mathbf{p}_k - h(\mathbf{p}_j, \mathbf{p}_i, \mathbf{p}_k) \|^2}{N(A_{\mathbf{p}_i})}, \tag{3}$$

where $h(\mathbf{p}_j, \mathbf{p}_i, \mathbf{p}_k)$ is the 3D position of the intersection of the aligned surface with the normal line for vertex $\mathbf{p}_k \in A_{\mathbf{p}_i}$. By normal line for p_i, we mean the line passing through $\mathbf{p}_i$ in the direction of normal vector for $\mathbf{p}_i$. $N(A_{\mathbf{p}_i})$ is the number of vertices in spherical patch $A_{\mathbf{p}_i}$. As shown in Fig. 2, intuitively $SSD(\mathbf{p}_i, \mathbf{p}_j)$ represents the distance between all neighboring points to $\mathbf{p}_i$, namely $\mathbf{p}_k$, and their projection onto the aligned surface corresponding to point $\mathbf{p}_j$ in the data region.

Similarly, texture similarity SSD_t is defined as the sum of difference in color between vertices in patch $A_{\mathbf{p}_i}$ and texture on aligned surface around $\mathbf{p}_j$ as follows:

$$SSD_t(\mathbf{p}_i, \mathbf{p}_j) = \sum_{\mathbf{p}_k \in A_{\mathbf{p}_i}} \frac{\| I(\mathbf{p}_k) - I(h(\mathbf{p}_j, \mathbf{p}_i, \mathbf{p}_k)) \|^2}{N(A_{\mathbf{p}_i})}, \tag{4}$$

2.2. Energy minimization

After initializing vertices' positions and colors in the missing region as in step (b), they are iteratively updated in step (c) so as to minimize E_s and E_t jointly. Step (c) consists of five processes. In the first three processes, we calculate the optimal positions of vertices in Ω so as to minimize E_s, and in the second two processes, we calculate the optimal colors of vertices in Ω so as to minimize E_t. Specifically, the following processes are iterated until convergence: (c-1) parallel search for similar shape patches; (c-2) parallel update of positions of all vertices in Ω; (c-3) inserting and merging of vertices in Ω; (c-4) parallel search for similar texture patches; (c-5) parallel update of colors of all vertices in Ω.

In practice, the update of vertex positions for minimizing E_s influences energy E_t and vice versa. However, such influences are decreased with iterations as each energy term converges. In the following subsections, each process is described in more detail.

2.2.1. *Minimizing energy for shape*

This subsection describes the method for minimizing energy E_s for shape, covering processes (c-1) through (c-3). In process (c-1), a data region is searched for the most similar shape patch keeping positions and colors of vertices in Ω fixed. The position $\hat{\mathbf{p}}_i$ of the shape patch most similar to the patch around

$\mathbf{p}_i \in \Omega'$ is determined by the following equation:

$$\hat{\mathbf{p}}_i = \underset{\mathbf{p}_m \in \Phi}{\operatorname{argmin}}(SSD_s(\mathbf{p}_i, \mathbf{p}_m) + \alpha SSD_t(\mathbf{p}_i, \mathbf{p}_m)). \quad (5)$$

Here, correlation between shape and texture is taken into account by calculating not only shape similarity but also texture similarity.

In process (c-2), the positions of all vertices in the missing region are updated in parallel in the similar way to [10]. In the following, we briefly describe the method for calculating the position of vertex $\mathbf{p}_i$ when the position of the most similar shape patch is known. Here, SSD_t in E_s can be ignored because colors of vertices are fixed. Energy E_s can be decomposed into element energy $E_s(\mathbf{p}_i)$ ($\forall \mathbf{p}_i \in \Omega$) which includes only position $\mathbf{p}_i$ as a parameter:

$$E_s(\mathbf{p}_i) = \sum_{\mathbf{p}_k \in A_{\mathbf{p}_i}} \frac{w_{\mathbf{p}_k}}{N(A_{\mathbf{p}_i})} \parallel \mathbf{p}_i - h(\hat{\mathbf{p}_k}, \mathbf{p}_k, \mathbf{p}_i) \parallel^2 . \quad (6)$$

The relationship between energy E_s and element energy $E_s(\mathbf{p}_i)$ can then be expressed as follows:

$$E_s = \sum_{\mathbf{p}_i \in \Omega} E_s(\mathbf{p}_i) + C, \quad (7)$$

where C is constant denoting the energy for the vertices in region $\overline{\Omega} \cap \Omega'$; this is because the positions of vertices are fixed and known in this region in the process (c-2). Our approach to minimize E_s is to minimize each element energy $E_s(\mathbf{p}_i)$ independently. To do so, we note in Fig. 3 that all the corresponding points $h(\hat{\mathbf{p}}_k, \mathbf{p}_k, \mathbf{p}_i)$ ($\forall \mathbf{p}_k \in A_{\mathbf{p}_i}$) exist on the normal line for vertex $\mathbf{p}_i$ because of the definition of $h(\hat{\mathbf{p}}_k, \mathbf{p}_k, \mathbf{p}_i)$. Therefore, assuming the normal vector of $\mathbf{p}_i$ does not change after updating the positions of neighboring points to $\mathbf{p}_i$, the position $\mathbf{p}_i$ that minimizes $E_s(\mathbf{p}_i)$ can be approximated as follows:

$$\mathbf{p}_i \approx \sum_{\mathbf{p}_k \in A_{\mathbf{p}_i}} w_{\mathbf{p}_k} h(\hat{\mathbf{p}}_k, \mathbf{p}_k, \mathbf{p}_i) / \sum_{\mathbf{p}_k \in A_{\mathbf{p}_i}} w_{\mathbf{p}_k}. \quad (8)$$

In practice, the value in Eq. (8) is only an approximate solution to minimize $E_s(\mathbf{p}_i)$ because the normal direction does change due to the update of the positions of vertices. Nevertheless, a reasonably good solution can be obtained as the energy converges because the change in the normal vector becomes smaller as the energy converges.

In process (c-3), the spatial distribution of vertices in Ω is made uniform by inserting and merging vertices while taking into account their density; this is because the update of vertex positions in process (c-2) biases the spatial distribution. In particular, we employ upper and lower thresholds for the length of a side of a triangle. If a side is longer than the upper threshold, a vertex is added to the midpoint of the side. The color of the new vertex is calculated by averaging colors of two vertices on the side. If a side is shorter than the lower threshold, two vertices on the side are merged and the new vertex is put on the midpoint of the side. The color is the average of colors of the two vertices.

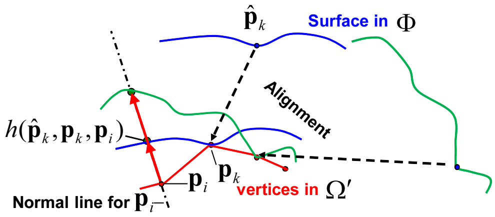


Fig. 3. Corresponding points on the normal line for vertex $\mathbf{p}_i$.

2.2.2. *Minimizing energy for texture*

After the update of vertices' positions in Ω, colors of vertices in Ω are updated so as to minimize energy E_t for texture, covering processes (c-4) and (c-5). In process (c-4), a data region is searched for the most similar texture patch keeping positions and colors of vertices in Ω fixed. Here, correlation between shape and texture is taken into account as well as searching for similar shapes. The position $\tilde{\mathbf{p}_i}$ of the texture patch most similar to the texture around $\mathbf{p}_i \in \Omega'$ is determined by the following:

$$\tilde{\mathbf{p}}_i = \underset{\mathbf{p}_n \in \Phi}{\operatorname{argmin}}(SSD_t(\mathbf{p}_i, \mathbf{p}_n) + \beta SSD_s(\mathbf{p}_i, \mathbf{p}_n)). \quad (9)$$

In process (c-5), the color of all vertices in the missing region are updated in parallel using the same idea as in process (c-2). Here, SSD_s in E_t can be ignored. Energy E_t can also be decomposed into element energy $E_t(\mathbf{p}_i)$ as follows:

$$E_t(\mathbf{p}_i) = \sum_{\mathbf{p}_k \in A_{\mathbf{p}_i}} \frac{\lambda_{\mathbf{p}_k}}{N(A_{\mathbf{p}_i})} \parallel I(\mathbf{p}_i) - I(h(\tilde{\mathbf{p}}_k, \mathbf{p}_k, \mathbf{p}_i)) \parallel^2 . \quad (10)$$

Therefore, color I($\mathbf{p}_i$) that minimizes $E_t(\mathbf{p}_i)$ is calculated as follows:

$$I(\mathbf{p}_i) = \sum_{\mathbf{p}_k \in A_{\mathbf{p}_i}} \lambda_{\mathbf{p}_k} I(\mathbf{h}(\tilde{\mathbf{p}}_k, \mathbf{p}_k, \mathbf{p}_i)) / \sum_{\mathbf{p}_k \in A_{\mathbf{p}_i}} \lambda_{\mathbf{p}_k}. \quad (11)$$

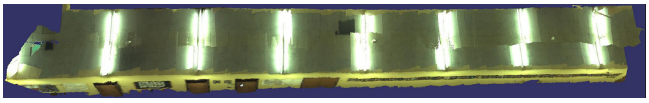
2.2.3. *Coarse-to-fine approach*

In order to efficiently complete the missing regions and avoid local minima, a coarse-to-fine approach is employed for energy minimization. Here, every time the energy converges, radius l of spherical patch A_{p_i} is decreased, and the number of vertices in region Ω' used for calculating energy is increased by changing the thresholds in process (c-3) for density of vertices. In addition, a pyramid for the texture image is generated offline and the resolution of a texture image is increased from the coarse to fine.

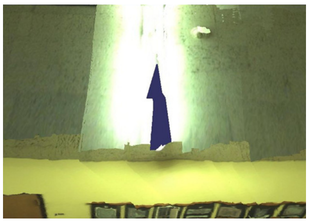
3. EXPERIMENTS

To demonstrate the effectiveness of the proposed method, we have applied the method to two models: an indoor model with 453,281 vertices which is created using indoor localization and modeling algorithm [11] as shown in Fig. 4(a), and a

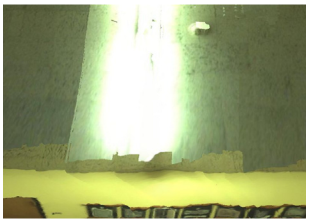
(a) Indoor model with holes



(b) Completed model



(c) Close-up of a hole



(d) Close-up of a completed hole

Fig. 4. Completion for a indoor model with holes.

(a) Bowl model with a hole

(b) Completed model ($\beta = 0.5$)

(c) Completed model ($\beta = 0$)

(d) Completed model (b) with no texture

Fig. 5. Completion for a bowl model with a hole.

small bowl model with 4,680 vertices as shown in Fig. 5(a). A missing region of the bowl model is generated manually. In the experiments, the average position and color of the boundary vertices of the missing region are used as an initial vertex's position and color in the missing region. We use a PC with Core i7 1.6GHz CPU and 8GB of memory.

As for the completion of the indoor model, the entire model is divided into multiple partial sub-models each with only one hole. Then each hole is filled in individually. Fig. 4(b) shows the completed model, and Figs. 4(c) and 4(d) show the close-up of one of the holes before and after completion. From these figures, we can confirm that natural shapes and textures are generated in the holes. It took 13,868 seconds to complete the all holes for this model.

As for the completion for the bowl model which has a smooth curved surface and bumps along the texture edges, we show two kinds of results using different weights β for SSD_s in E_t. In the completed model of Fig. 5(b) with $\beta = 0.5$, texture edges are generated in the missing region. On the other hand, plausible edges are not generated in the completed model of Fig. 5(c) with $\beta = 0$. From this difference in results, we can confirm that taking into account correlation between shape and texture similarity is effective. However, the generated texture is slightly blurred because the resolution of texture is higher than the resolution of vertices. Finally, Fig. 5(d) shows the completed model in Fig. 5(b) without texture. As seen, natural shapes are generated in the missing region. It took 512 seconds to complete the hole for this model.

4. CONCLUSIONS

In this paper, we have presented a novel method for surface completion of shape and texture based on energy minimization and demonstrated its effectiveness experimentally on two datasets. Future work involves increased resolution of generated texture. In addition, our method can be applied to 3D models for large environments such as outdoor scenes.

5. REFERENCES

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